# What are the most useful things to learn in math?

## (also known as "Why can't I use a calculator?")

**A brief look at research on the use of calculators in the classroom**

Recently, an adult learner asked the question that has been asked many times before: “Can I use a calculator to do this math?”

I am embarrassed to say that I have never taken the time to properly answer the question by seriously looking at the arguments for and against the use of calculators. It was now time to do that.

I started by Googling “calculators in the classroom”. Quickly, however, the material I read led me from the question of calculators in the classroom to the bigger and more important question of “ What are the skills people need to be able to learn and use math?” After reading and learning more about the skills people need to learn and use math, the answer to the question of “Can I use a calculator” became clear. And the answer surprised me.

### What are the skills people need to be able to learn and use math?

The National Council of Teachers of Mathematics[1] separates the skills needed to learn and use math into 3 “senses”:

1. Number Sense
2. Operation Sense
3. Computation Sense

These “senses” are defined as follows:

**Number Sense** is what helps a person “understand numbers, ways of representing numbers, relationships among numbers, and number systems”[2] It involves:

- Understanding ways of representing numbers
  - 102 is 1 hundred, 0 tens and 2 ones
  - 0.25 has no wholes, 2 tenths and 5 hundredths
  - ⅜ means the whole is broken into 5 parts and we are interested in 3 of them
  - ½ = 0.5 = 50%

- Understanding relationships between numbers and making comparisons
  - 0.25 has 2 tenths *AND* 0.5 has 5 tenths *SO* 0.5 is more than .25
  - ⅜ is more than one half *BECAUSE* 3 is more than half of 5
  - 1 ½ = 3/2

[1] The National Council of Teachers of Mathematics (NCTM) is the world's largest mathematics education organization throughout the United States and Canada. This info available at https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Number-and-Operations/

[2] “Principles and Standards: Number and Operations”, https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Number-and-Operations/

- Using numbers in flexible ways when adding, subtracting, multiplying or dividing
  Can convert fractions to decimals and then use a calculator

- Using strategies when counting, measuring or estimating.
  Grouping numbers to add more quickly...

- Rounding numbers

Lack of number sense leads to
- 1/5 + 2/5 = 3/10 because 1+2=3 and 5+5=10
- 0.2 + 4 = 6 because 2+4=6
- 0.25 > 0.5 because 25 is bigger than 5

**Operation Sense** is what helps a person "understand meanings of operations and how they relate to one another."[3] It involves, among other things, knowing that

➕ **Addition:**
*Combines* things (opposite of subtracting)
You can only add things that are the same

➖ **Subtraction:**
*Separates* things (opposite of adding)
Used to take away an amount to get a smaller amount
Used to find the difference between two numbers to make a comparison
You can only subtract things that are the same

❌ **Multiplication:**
*Combines* many things when they are all the same
Shortcut to adding the same thing over and over

➗ **Division:**
*Separates* an amount into equal parts (or finds how many times one amount fits into another)

**Computation Sense** is what helps a person decide the appropriate method of calculation and make quick, accurate and reliable use of that method. This involves:
- Awareness of different methods of calculation (mental, paper and pencil, calculator)
- Ability to decide which of the three methods to use based on which in a given situation is the fastest and most reliable and accurate.
- Mastery of the methods used

Computation Sense returns us to the question of the methods we use to compute answers. But the question is now in the broader context of the other senses (Number and Operation) that are essential for learning and using math.

## The methods of computation

There are 3 methods for doing calculations with numbers: Mental math, pencil and paper, and calculator. Each has its place.

[3] "Principles and Standards: Number and Operations", https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Number-and-Operations/

**Comparing the 3 methods by speed, accuracy and reliability of calculation**

It has been pointed out that "when there is more than one [method] with sufficient power to complete a particular task, the [method] of choice is normally the faster, or the more accurate, or the more reliable." [4] The three methods of calculating, then, can be compared by speed, accuracy and reliability of calculation:

1. Mental math: for the small and simple numbers
   - often quickest for adding and subtracting simple or small numbers
   - often quickest for × and ÷ questions found in 1x1 to 10x10 times tables [for one and two digit calculations] [5]
   - often quickest for estimating
   - reliable and accurate if there is a
     - strong number sense
     - strong operation sense
     - strong recall of times tables and number facts.

2. Pencil and paper math
   - slower than mental for simple numbers
   - slower than calculator for larger numbers
   - reliable and accurate *if* there is a
     - strong number sense
     - strong operation sense
     - strong recall of times tables and number facts
     - AND
     - strong recall of algorithms of long division, fraction subtraction, ...

3. Calculator/computer math
   - slower than mental math for simple and small numbers
   - faster than pencil and paper for larger numbers
   - reliable and accurate *if*
     - correct use of calculator
     - strong number sense
     - strong operation sense

Note that all three methods are rooted in a strong number sense and a strong operation sense. Use of a calculator does not negate the importance of these senses. The use of the three senses above is essential to correct problem solving regardless of whether a calculator is used or not. This is because performing operations on numbers to calculate an answer (regardless of method) is only one small part of the larger process of solving a problem.

Use of a calculator does not eliminate the role of mental math. For estimates or calculations with simple numbers, mental math is often the method of choice because it can be faster than a calculator. The speed, accuracy and reliability of mental math depend on strong number sense and memorization of multiplication tables at least to 10x10.

[4] "Paper-and-Pencil Skills in a Calculator/Computer Age." Presented November 12 & 13, 1994 by UCSMP Director Zalman Usiskin. available at http://ucsmp.uchicago.edu/resources/conferences/1994-11-12/

[5] "Groping and Hoping for a Consensus on Calculator Use." Mathematics Education Dialogues, Volume 2, Issue 3 May/June 1999 found at oldmoodle.escco.org/file.php/1/MATH/Groping_Hoping.pdf

For almost every other situation, however, the calculator is the method of choice. At one time, it may have been true that pencil and paper calculation skills were important because calculators were not always available. Now, however, the situation is reversed. Because every cell phone is also a calculator, a calculator can often be found more readily than a pencil and paper.

The one significant difference between using calculators or pencil and paper math skills is that pencil and paper math skills require the learning and memorization of many simple and not-so-simple algorithms (steps) to perform the calculations.

In 1994, Zalman Usiskin presented a paper entitled "Paper-and-Pencil Skills in a Calculator/Computer Age." He stated:

> "There are some who claim that doing the paper-and-pencil algorithms trains the mind in good ways, that going through these procedures teaches orderliness and care. There has never been any evidence of this. Even mathematicians, who tend to be better at these algorithms than other people, are no more orderly than other people. ...
>
> The evidence is, in fact, to the contrary. These algorithms seem to take students' minds off of the bigger picture, and turn many students off towards mathematics. ..
>
> It is true that the idea of an algorithm is an important one — that people do need to follow directions in many life activities. The learning of mathematics may contribute to this, but probably no more than other subjects in school" [6]

**Comparing the 3 methods by extent to which they do not distract from higher level problem solving.**

This quote introduces a second way to evaluate the three methods of calculation - evaluating the extent to which each method allows the learner to focus energies on higher level problem solving without getting distracted or bogged down in the calculation of the numbers themselves.

When solving a math problem, the decision to use mental math is based, again, on speed, accuracy and reliability. Mental math will be used only if it is quick and correct. If a person deems the numbers to be too complex for quick mental math, they will choose another method. Because it is the method of choice only when quick and easy, it uses little time and energy and therefore creates little to no distraction from the problem at hand.

Use of pencil and paper to solve a problem requires the use of memorized algorithms with numbers that may be simple whole numbers or more complex decimals or fractions. Usiskin notes above that the not-so-simple algorithms required to do pencil and paper calculations may, in fact, be a distraction from the larger problem being solved. If the pencil and paper math requires use of longer algorithms

[6] Paper-and-Pencil Skills in a Calculator/Computer Age. Presented November 12 & 13, 1994 by UCSMP Director Zalman Usiskin. available at http://ucsmp.uchicago.edu/resources/conferences/1994-11-12/

(long division) or several different algorithms (fractions) then the time and effort spent on the calculation distracts from the larger picture of the problem that is being solved and the meaning of the answer we have calculated. The National Research Council stated "Since each person has a limited amount of mental effort that he or she can expend at any one time, more complex tasks can be done well only when some of the subtasks [require little to no mental effort]."[7]

Use of calculators, then, "gives[s] students an opportunity to manipulate larger numbers while solving higher level thinking investigations" [8]

Beyond simply allowing more focus on problem solving, use of a calculator has also been found to increase interest in math. "Current research evidence indicates that students who are given opportunities to work on their problem solving skills enjoy the subject more, are more confident and are more likely to continue studying mathematics, or mathematically related subjects, beyond the age of 16." [9]

## The last word

Perhaps the most rigorous exploration of calculator use in the classroom was performed by Aimee. J. Ellington of Virginia Commonwealth University. in "A Meta-Analysis of the Effects of Calculators on Students' Achievement and Attitude Levels in Precollege Mathematics Classes" [10], Ellington analyzed the findings of 54 studies of calculator use in the classroom. The analysis found:

1. Students' operational skills and problem solving skills improved when calculators were an integral part of testing and instruction
2. In all cases, calculator use did not hinder the development of mathematical skills
3. Students using calculators had better attitudes toward mathematics than their non calculator counterparts.

[7] "Adding it up: Helping Children learn mathematics" National Research Council, Washington DC, 2001 pg 351
[8] "Tools for Mathematical Understanding in Middle School" found in Groping and Hoping for a Consensus on Calculator Use. Mathematics Education Dialogues, Volume 2, Issue 3 May/June 1999 found at oldmoodle.escco.org/file.php/1/MATH/Groping_Hoping.pdf
[9] "What is a Mathematically Rich Task?" found at https://nrich.maths.org/6299
[10] "Meta-Analysis of the Effects of Calculators on Students' Achievement and Attitude Levels in Precollege Mathematics Classes." Journal for Research in Mathematics Education. 2003, Vol 34, NO5, 433-463 found at https://pdfs.semanticscholar.org/583a/a0edf4abe9a09f4df46b87620f42b2d59f54.pdf?_ga=2.180296996.1854923723.1550776355-884829056.1550776355

## Conclusion:

Here is what surprised me. I discovered that I had equated learning pencil and paper math skills with learning number sense and operation sense. I know now this is wrong. A person can spend weeks mastering the long division algorithm and still not know when to use division to solve a problem and still not recognize when an answer is not reasonable.

And similarly, use of a calculator does not replace strong number sense and operation sense. In fact, a calculator (like pencil and paper math skills) is useless without a strong number sense and operation sense.

So my conclusions are as follows:

- The focus of math upgrading should be problem solving and developing number sense, operations sense and computation sense.
- Although a useful method for situations where a calculator is not available (rare) or permitted (tests), teaching the algorithms of pencil and paper calculations should not be the default for math upgrading. In "Changing the way we teach math: A Manual for Teaching Basic Math to Adults", Kate Nonesuch says that teachers should "view computation as a tool for problem solving, not an end in itself" [11]
- Nothing can replace mental math as the method of choice for quickly working with simpler numbers and estimations. Therefore, knowledge of the times tables at least to 10x10 is important.
- Calculators are the tool of choice in most environments for quick, accurate and reliable calculations with larger or more complex numbers. They are ever-present in the form of a cell phone. As with mental math and pencil and paper calculations, they are useful for problem solving only when partnered with a strong number sense, operational sense.
- Use of a calculator frees learners to focus on higher level problem solving without getting distracted by focus on pencil and paper calculations.
- Learning pencil and paper methods of long division, … may be only necessary for people preparing for tests that do not allow a calculator or in a workplace where pencil and paper (or block of wood) is fastest.

[11] "Changing the way we teach math: A Manual for Teaching Basic Math to Adults" by Kate Nonesuch available at http://www.brown.edu/Departments/Swearer_Center/Literacy_Resources/mathman.pdf

## Other sources:

**National Council of Teachers of Mathematics:**
Founded in 1920, the National Council of Teachers of Mathematics (NCTM) is the world's largest mathematics education organization, with 60,000 members and more than 230 Affiliates throughout the United States and Canada. Interested in membership? https://www.nctm.org

**The University of Chicago School Mathematics Project (UCSMP):**
was founded in 1983 with the aim of upgrading and updating mathematics education in elementary and secondary schools throughout the United States. Since its inception it has been the largest university-based mathematics curriculum project in the United States. http://ucsmp.uchicago.edu

**NRICH**
NRICH is an innovative collaboration between the Faculties of Mathematics and Education at the University of Cambridge, part of the University's Millennium Mathematics Project.
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https://nrich.maths.org